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Abstract. We report the results of photon-counting experiments which demonstrate the factorization properties of the correlation functions, up to sixth order, of coherent and incoherent optical fields.

In his fundamental papers on the quantum theory of optical coherence Glauber (1963 a, b) constructed a sequence of correlation functions

$$G^{(n)}(\boldsymbol{r}_1t_1 \dots \boldsymbol{r}_nt_n, \boldsymbol{r}_{n+1}t_{n+1} \dots \boldsymbol{r}_{2n}t_{2n})$$

for the electric-field vectors $\mathscr{E}(\boldsymbol{r}, t)$. For a fully coherent field, such as might be obtained from an ideal laser, he showed that the higher-order correlation functions factorize as follows:

$$G^{(n)}(\boldsymbol{r}_1t_1 \dots \boldsymbol{r}_{2n}t_{2n}) = \prod_{i=1}^{n} \mathscr{E}^*(\boldsymbol{r}_it_i) \prod_{j=n+1}^{2n} \mathscr{E}(\boldsymbol{r}_jt_j) \tag{1a}$$

while for an ideally incoherent (first-order coherent Gaussian) field he gave the relations

$$G^{(n)}(\boldsymbol{r}_1t_1 \dots \boldsymbol{r}_{2n}t_{2n}) = n! \prod_{i=1}^{n} \mathscr{E}^*(\boldsymbol{r}_it_i) \prod_{j=n+1}^{2n} \mathscr{E}(\boldsymbol{r}_jt_j). \tag{1b}$$

In this letter we present experimental results which verify both these formulae up to sixth order for the case when the $2n$-space coordinates coincide.

The measurements were made by photon counting with a single detector which was effectively an ideal broadband device of negligible spatial extension and quantum efficiency α. Equations (1*a*) and (1*b*) then reduce, respectively, to

$$G^{(n)}(t_1 \dots t_n, t_n \dots t_1) = \prod_{j=1}^{n} G^{(1)}(t_j, t_j) \tag{2a}$$

and

$$G^{(n)}(t_1 \dots t_n, t_n \dots t_1) = n! \prod_{j=1}^{n} G^{(1)}(t_j, t_j). \tag{2b}$$

The higher-order correlation functions are related to the photon-counting distribution $p(m, T)$ from the detector through its factorial moments (the actual moments of the intensity-fluctuation distribution), defined by (Glauber 1965, equation (17.12))

$$N^{(r)}(T) = \sum_{m=r}^{\infty} m(m-1) \dots (m-r+1)p(m, T) \tag{3}$$

where T is the sampling time. For small T this relationship is, by integration of equation (17.24) of Gauber (1965),

$$\alpha^n G^{(n)}(t_1 \dots t_n, t_n \dots t_1) = N^{(n)}/T^n. \tag{4}$$

In terms of the normalized factorial moments given by

$$n^{(r)} = \frac{N^{(r)}}{(N^{(1)})^r} \tag{5}$$

the relations (2*a*) and (2*b*) take the forms, respectively,

$$n^{(r)} = 1 \tag{6a}$$

$$n^{(r)} = r! \tag{6b}$$

The best fields for such experiments have been found in the course of our work to be produced by a He–Ne single-frequency temperature-stabilized laser (Spectra Physics model 119) for the coherent source, and by the same laser light scattered from small spherical particles undergoing Brownian motion for the incoherent source. Details of the experimental arrangements have been given previously (Johnson *et al.* 1966 b, Jakeman *et al.* 1968).

The photon-counting distributions are obtained from a total number M of samples, which is limited by the speed of the electronic equipment and by the overall time of the experiment. The statistical accuracy of the corresponding factorial moment, for a fixed overall time, improves as M increases for the coherent field, but reaches a maximum independent of M for the incoherent field. A study of the statistical properties of periodic sampling in connection with photon-counting experiments will be published elsewhere, but we quote here the results relevant to this paper. Let $\mathcal{N}^{(r)}$ be the estimator of $N^{(r)}$ obtained from M periodic samples; then, writing $N^{(1)} = \bar{n}$, we have for the coherent field

$$\operatorname{var}(\mathcal{N}^{(r)}) = \frac{\bar{n}^{2r}}{M} \sum_{s=1}^{r} \frac{s!}{\bar{n}^s} \binom{r}{s}^2 \tag{7a}$$

and for the incoherent field

$$\operatorname{var}(\mathcal{N}^{(r)}) = (r!)^2 \frac{\tau_c}{\tau} \frac{\bar{n}^{2r}}{M} \sum_{s=1}^{r} \frac{1}{s} \binom{r}{s}^2 ; \tag{7b}$$

τ_c is the coherence time of the incoherent source, τ is the period between samples, and for the formulae to be valid we must have $\tau_c/r\tau$ much greater than unity. We note that the formula given by Arecchi *et al.* (1966, equation (7)) for the variance of the actual moments of $p(m, T)$ is incompatible with our results. For reasonable experimental times (~ 10 min) it is possible to obtain accuracies better than a few per cent in the first six moments for the coherent source. In table 1 we show the result of a single experiment,

Table 1. Normalized factorial moments for coherent source

Normalized factorial moment	10^7 samples	
	Experiment	Theory
$n^{(1)}$	1·0000	1·0000
$n^{(2)}$	1·0002	1·0000 ± 0·0009
$n^{(3)}$	1·0001	1·0000 ± 0·0017
$n^{(4)}$	1·0006	1·0000 ± 0·0030
$n^{(5)}$	1·003	1·000 ± 0·012
$n^{(6)}$	0·994	1·000 ± 0·036

corrected for dead-time effects (Johnson *et. al.* 1966 a) for the coherent source with 10^7 samples each of 1 μs.† For the incoherent source the errors are much higher owing to the effect of a long coherence time (0·145 s) in equation (7*b*), and in table 2 we show the results

Table 2. Normalized factorial moments for incoherent source

Normalized factorial moment	10^5 samples		9×10^6 samples	
	Experiment	Theory	Experiment	Theory
$n^{(1)}$	1·000 ± 0·036	1·000 ± 0·038	1·000	1·000
$n^{(2)}$	2·00 ± 0·15	2·00 ± 0·16	2·004	2·000 ± 0·017
$n^{(3)}$	6·05 ± 0·85	6·00 ± 0·85	6·05	6·00 ± 0·09
$n^{(4)}$	24·7 ± 5·9	24·0 ± 5·7	24·7	24·0 ± 0·6
$n^{(5)}$	130 ± 45	120 ± 49	130	120 ± 5
$n^{(6)}$	805 ± 461	720 ± 510	805	720 ± 54

† Similar results were presented at the Conference CI07 in Paris, May 1966, but have not previously been published.

of 90 experiments, each of 10^5 samples of 1 ms; dead-time effects are now unimportant. For the latter experiments τ/T was 1, T/τ_c was 7×10^{-3} and the ratio of the detector area to the coherence area was 4×10^{-3}. The variance (scaled by the normalization) observed over the 90 experiments can be seen to agree well with that predicted by equation (7*b*), and the mean values are in satisfactory, although not perfect agreement, in both limits with equations (6).

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